

PUPT-2244
hep-th/yymmnnn

De Sitter Space and Eternity

A.M. Polyakov

Joseph Henry Laboratories
Princeton University
Princeton, New Jersey 08544

Abstract

This paper explores infrared quantum effects in the de Sitter space. The notion of "eternal manifolds" is introduced and it is shown that in most cases the de Sitter space doesn't belong to this class. It is unstable under small perturbations which may cause a breakdown of the de Sitter symmetry. The de Sitter string sigma model is discussed. It is argued that the gauge theory at the complex coupling is dual to the matrix elements of vertex operators in the de Sitter space, taken between the Bunch - Davies vacuum and the "out" state without particles. The described infrared effects are likely to screen away the cosmological constant.

September 2007

1 Introduction.

Many years ago I conjectured [1] that the cosmological constant may be screened by the infrared fluctuations of the metric, much like the electric charge in quantum electrodynamics. This effect, if it exists, must be non-perturbative, related to the fluctuations of the metric $g_{\mu\nu}$ near zero, and not near a classical background value. In the time- dependent picture the screening is equivalent to the instability of space- times with constant curvature. In this picture the initially present curvature is gradually decaying. It is important, therefore to find out, whether the de Sitter space carries the infrared seeds of its own destruction. This is the topic of the present article.

It is not straightforward to find an appropriate framework for discussing this question. The notion of fields and particles in the curved Lorentzian backgrounds is ambiguous and the Hamiltonian may not exist at all. We have to formulate a principle which describes quantum field theories in these circumstances. With this goal in mind we will postulate that a free particle on a stable manifold must propagate with an amplitude

$$G_{++}(x, x') = \sum_{(P_{xx'})} e^{-imL(P_{xx'})} \quad (1)$$

where the sum goes over all paths belonging to the manifold and connecting the points x and x' , $L(P)$ is the length of the path , m is the mass of the particle and the subscripts will be explained later. When we consider interacting particles, the Feynman prescription is to draw the diagrams and to integrate the position of the vertices over the whole space-time.

That seems to contradict causality, since the observables at a given moment of time seem to depend on the interaction in the future. The resolution of this puzzle is well known. The use of Feynman's diagrams presupposes that the vacuum is stable , that nothing new will ever happen. This assumption restores the causality and leads to CPT symmetry. In this case the manifold is eternal and the future is being prefixed. We define (perturbatively) " *eternal manifolds* " as the ones for which the Feynman rules are valid and the vacuum loops do not contain an imaginary parts.

If, however, the vacuum is unstable, the picture is quite different. To calculate the present we must eliminate all the signals from the future. The way to do it was discovered by Schwinger [3] and developed by a number of people [4]. According to [3], we must double the manifold and consider the two copies (+/-) with the chronological ordering from $-\infty$ to ∞ on the first sheet and back on the second. The vertices of Feynman's diagrams are now labeled by their position x and an Ising variable $\sigma = +/-$ indicating the choice of the sheet.

The *eternity test* is the condition that all diagrams in which a particle changes the sheet (thus containing the G_{+-} Green function) are zero. A slightly different way to put it is to require that all diagrams containing "spiders" - a (-) point surrounded by (+) -s only are zero. A spider represents spontaneous particle creation from the vacuum. If this is forbidden we return to the ordinary Feynman's diagrams. We will explore this condition below and find that in the de Sitter space in the certain cases it is violated. The next step will be to understand this from the point of view of gauge/strings duality.

If de Sitter space turns out to be intrinsically unstable, it must be described by a *non-unitary conformal field theory*. A natural candidate for such a theory would be a Yang-Mills theory with complex coupling obtained by the analytic continuation from negative to positive curvature. We also should expect to find a description in terms of the world-sheet sigma model with the de Sitter space as a target.

These statements are puzzling since the de Sitter space has been investigated in many works and no instability of the so called Bunch - Davies vacuum was ever found (see [13], [9] for the excellent reviews). My assertion here is that neither this nor any other vacuum passes the "eternity test". It is perhaps appropriate to explain the meaning of this test, leaving technical details (discussed below).

Let us consider the Bunch - Davies vacuum for a scalar field φ . This field in the de Sitter space with the standard metric $ds^2 = \cosh^2 t d\Omega^2 - dt^2$ can be expanded in partial waves. Each partial wave φ_m (m being angular momentum, see below) can be further expanded as $\varphi_m = a_m \phi_m(t) + a_m^\dagger \phi_m^*$, where ϕ -s are the appropriate solutions of the Klein-Gordon equation (27) and a, a^\dagger are the creation and annihilation operators. The Bunch- Davies (aka Hartle- Hawking, aka Euclidean) vacuum E is defined as $a|E\rangle = 0$. There are also two other vacua $|\pm\rangle$, defined with the modes without positive frequencies in the future or negative in the past (we will also call them "in"

and "out" states when it is not ambiguous). These are the states without particles at the past / future infinity respectively. They have the feature $|\langle E|\pm\rangle| < 1$, meaning that in the Euclidean vacuum particles are created. By itself this is not yet an instability. One might think that the Euclidean vacuum is eternal, but viewed from infinity, contains particles. Until we include interactions this is a possible point of view.

Let us now take, say, the φ^4 interaction and turn it on adiabatically and then turn it off in the same way. In the usual field theory the vacuum will acquire a phase after this procedure but otherwise will remain unchanged. Not so in the de Sitter space. From the formulae given below it follows that $\langle E|T \exp(-i \int \lambda(\epsilon t) \varphi^4(dx)|E\rangle \rightarrow 0$ as $\epsilon \rightarrow 0$. In other words, with any interaction $|E\rangle_{out}$ is orthogonal to $|E\rangle_{in}$. Technically this "*adiabatic catastrophe*" occurs since we can evaluate the above matrix element by the usual Feynman diagrams, and, as we will see, they contain very strong infrared divergences.

One can try to study instabilities in a different way, by looking at the *in/in* matrix elements and using the Schwinger prescriptions. This is a kind of a "*laissez - faire*" approach when we let the system develop by itself. In this framework the instability means that small perturbations breaking the de Sitter symmetry grow with time, while the divergences, requiring the IR cut-off (like ϵ in the above formula) go away. It is reasonable to expect that as a result of these infrared phenomena, a small perturbations of the metric and other fields will be amplified with time, destroying the de Sitter symmetry. This can be viewed as a kind of spontaneous symmetry breaking (with large value of time playing the role of the large volume in statistical mechanics). We must stress, however, that there is still no complete proof of the instability in the Schwinger framework. It remains to be proved that in this case the vacuum polarization will lead to some kind of gravitational instability. Below we will give some arguments in favour of this statement. A closely related phenomenon happens in the Starobinski model [10] where the instability occurs due to the trace anomaly.

Another limitation of the present work is that it treats only interacting scalar fields and leaves aside the tensor structure and gauge fixing of the gravitational oscillations. Hopefully the qualitative effects we are after will still be present in the full theory. The "dS/CFT correspondence" has been discussed in the papers [2],[8],[11]. While the approaches of [2],[8] are quite different from the present paper, the ideas of [11] go in the same direction. Still, our conclusions below seem new. Many formulae for the various propagators used below must be well known to the experts in the field. Infrared

divergences have also been investigated before [5] [7], but we add some new results and approaches. The relation between infrared divergences and stability and the relation to the non-unitary gauge theory was discussed in [12] on a qualitative level; see also a recent discussion of the Euclidean infrared problems [6] . The particle production , instabilities and back reaction have been discussed from the various points of view in the papers [14] , [16] , [15] and references therein. A runaway particle production was studied in an interesting paper [17]

2 The composition principle and the Unruh detector

The formula (1) implies the asymptotic condition which uniquely fixes the propagator

$$G(x, x') \sim e^{-imL(x, x')} \quad (2)$$

where $L(x, x') \rightarrow \infty$ being the geodesic time-like length. Another way of putting it is to select the Green function of the Klein - Gordon equation which, for the time-like separations is analytic in mass if $\text{Im } m < 0$.

It is important to realize that the generic propagator would contain a superposition $Ae^{imL} + Be^{-imL}$. This propagator would violate the following composition principle which lies at the foundation of quantum field theory. We should be able to glue together two paths and obtain a single one. This condition is equivalent to unitarity in the ordinary quantum field theory. In the classical limit this gluing procedure is expressed by the relation $L(x, x') = L(x, y) + L(y, x')$ where y must extremize this expression. In the quantum case one has to account for the entropy on the world line (or the world sheet in the case of strings) and the relation is

$$\int dy G(x, y) G(y, x') = \sum_{(P_{xx'})} L(P) e^{-imL(P)} \sim \frac{\partial}{\partial m} G(x, x') \quad (3)$$

As a side remark, let us notice that in the case of strings the extra factor L in this formula is replaced by the free energy of the self-avoiding path belonging to the surface. These gluing relations in the flat space play the role of unitarity conditions for the corresponding field theory. It seems reasonable to postulate that the "right" propagators must respect these conditions in

the general case. If the two exponents are present, we would get in the asymptotic the term $\widetilde{L}(x, x') = L(x, y) - L(y, x')$, which, being minimized with respect to y , doesn't reproduce the left hand side.

The above condition has a curious relation to the behaviour of the Unruh detector. As well known [13], the probability for absorbing energy ε by this detector is given by

$$w(\varepsilon) \sim \int ds_1 ds_2 e^{-i\varepsilon(s_1 - s_2)} G(x(s_1), x(s_2))$$

where $x(s)$ is a trajectory of the detector and s is its proper time. Let us assume that the trajectory is a geodesic. In this case $G \sim Ae^{-im(s_1 - s_2)} + Be^{im(s_1 - s_2)}$. We see that the B coefficient, which violates the composition, simultaneously defines the absorption probability of the inertial detector. If it is non-zero, *the inertial detector can borrow energy from the vacuum indefinitely!* It is not surprising that such a vacuum won't live long.

Let us now turn to the de Sitter space. The standard approach uses the propagators obtained by analytic continuation from the sphere (which define the Bunch-Davies or Euclidean vacuum). Let us have a look at them, using for simplicity the 2d sphere. The Green function is given by

$$G(n, n') = \sum_{l=0}^{\infty} \frac{(2l+1)P_l(n \cdot n')}{l(l+1) + M^2} = \frac{\pi}{\sin \pi \nu} P_{\nu}(-n \cdot n') \quad (4)$$

here n is a unit vector, ν is defined by a relation $M^2 = -\nu(\nu + 1)$ and the above formula is obvious since the left hand side has the same poles and residues as the right one. It also has the expected logarithmic singularity at the coinciding points $(n \cdot n') = 1$. On a sphere it satisfies the composition principle

$$\frac{1}{2\nu+1} \frac{\partial}{\partial \nu} \left(\frac{1}{\sin \pi \nu} P_{\nu}(-n \cdot n') \right) = \frac{\pi}{\sin^2 \pi \nu} \int dn_1 P_{\nu}(-n \cdot n_1) P_{\nu}(-n_1 \cdot n') \quad (5)$$

For the future references, let us point out that in the $d+1$ dimensional case the only changes in these formulae are

$$P_{\nu}(z) \Rightarrow C_{\nu}^{\frac{d}{2}}(z); l(l+1) \Rightarrow l(l+d); 2l+1 \Rightarrow 2l+d \quad (6)$$

(where C are the Gegenbauer polynomials). We will obtain the dS/AdS spaces by analytic continuation from the sphere. But the procedure is non-trivial and is discussed in the next section

3 Analytic connections of various spaces.

The de Sitter space (or imaginary Lobachevsky space) is obtained from a sphere by an analytic continuation $n_0 \Rightarrow in_0$ and forms a one sheeted hyperboloid

$$n_k^2 - n_0^2 = 1 \quad (7)$$

The metric has the form $ds^2 = (dn_k)^2 - (dn_0)^2$. Since even complex change of variables doesn't change the scalar curvature, it remains constant and positive. The distance L between two points is given by the formula $\cosh L = (n \cdot n')$ and could be both time-like (real L) and space-like (imaginary L). The Klein-Gordon (or Laplace) equation, defining a propagator is also unchanged as we make the analytic continuation. Therefore we can obtain propagators on the de Sitter space from the propagator on a sphere by the simple substitution. The only problem is that a physical propagator in one case may be unphysical in another.

The Euclidean vacuum is defined by the propagator (4) in which we substitute $(n \cdot n') = (n_k n'_k) - n_0 n'_0$. This is a nice, well defined function with the expected singularity. However, it doesn't satisfy our additivity requirement. Indeed, in the limit of large L we get the following asymptotics, using the formulae $P_\nu(z) \sim Az^\nu + Bz^{-\nu-1}$ (at large z) and $\nu = -\frac{1}{2} + i\mu$; $M^2 = \frac{1}{4} + \mu^2$

$$G(n, n') \sim Ae^{-i\mu L} + Be^{i\mu L} \quad (8)$$

So this Green function is unlikely to be the right starting point for the quantum theory in the de Sitter space. The formula (5) is destroyed by the analytic continuation because on a hyperboloid it becomes divergent.

A different Green function which does satisfy our requirement is given by

$$G(n, n') = Q_\nu(n \cdot n') \quad (9)$$

It contains only one exponential in the time-like asymptotics and has a correct singularity at the coinciding points. But it also has an unexpected singularity at the antipodal points defined by $(n \cdot n') = -1$. Strange as it is, this singularity must appear if we define the propagator as a sum over trajectories. Indeed, the antipodal points can be connected by infinite number of geodesics. The functional integral includes the integration over the zero mode describing the subspace of these geodesics. If this subspace is non-compact, one expects and gets the divergence at the antipodal points. We will discuss this singularity below.

Before discussing the physical consequences of this propagator, let us analyze the AdS spaces. We can get them from a sphere by the continuation $n_k \Rightarrow in_k$, so that in this case we have a two-sheeted hyperboloid, $n_0^2 - n_k^2 = 1$. The important difference from the above is that we also have to change the sign of the metric on a sphere, $ds^2 \Rightarrow -ds^2$. In this way we obtain the AdS space with the euclidean signature (EAdS, or a real Lobachevsky space). Its metric is given by

$$ds^2 = (dn_k)^2 - (dn_0)^2 = y^{-2}((dx)^2 + (dy)^2) = (d\rho)^2 + \sinh^2 \rho (d\omega)^2 + \cosh^2 \rho (dt)^2 \quad (10)$$

where we introduced for the future use two standard parametrizations of the AdS. The scalar curvature $R = g^{\alpha\beta} R_{\alpha\beta}$ changes sign under the above transformation $g_{\alpha\beta} \Rightarrow -g_{\alpha\beta}$ and hence the space has constant negative curvature. More interestingly, under this transformation the covariant Laplacian also changes sign. Hence we have the following relation between the Green functions

$$G_{dS}(n, n', M^2) = G_{AdS}(n, n', -M^2) \quad (11)$$

This formula implies that the positive mass in the de Sitter space corresponds to a tachionic mass in AdS. That give us a first hint that the dS space may be unstable. However these analytic relations don't tell us which propagators are physical. This is determined by their spectral properties.

4 Spectral features of various propagators

We see that the propagators in the spaces of constant curvature are analytically related. Let us begin to analyze their spectral properties with the EAdS space. In the Poincare coordinates we have the following expression

$$z = (n \cdot n') = 1 + \frac{(x - x')^2 + (y - y')^2}{2yy'} = 1 + \frac{1}{\sin \sigma_1 \sin \sigma_2} (\cosh(t_1 - t_2) - \cos(\sigma_1 - \sigma_2)) \quad (12)$$

where we introduce variable $0 \leq \sigma \leq \pi$, $\cosh \rho = \frac{1}{\sin \sigma}$. This variable appears when the Poincare upper half-plane is conformally mapped onto an infinite strip. We see that since $z \geq 1$ and we must make a change $M^2 \Rightarrow -M^2$ when passing from a sphere. The only propagator which doesn't blow up in this space is

$$G(n, n') = Q_{-\frac{1}{2}+\mu}(z) \quad (13)$$

where $M^2 = -\frac{1}{4} + \mu^2$. Its spectral decomposition is given by the formula

$$G(n, n') = \int_0^\infty \lambda \tanh \pi \lambda \frac{P_{-\frac{1}{2}+i\lambda}(n \cdot n')}{\lambda^2 + \mu^2} \quad (14)$$

This is a precise analogue of (4) in which the angular momentum l is set to be $-\frac{1}{2} + i\lambda$. It is also instructive to look at the massless case, $\mu = \frac{1}{2}$. Since the metric (10) is conformally flat, the propagator in this case is just that of a free particle moving in the upper half plane or , equivalently, in a strip with the Dirichlet boundary conditions

$$G(n, n') = \log\left(\frac{z+1}{z-1}\right) = \sum_1^\infty \frac{1}{k} e^{-k|t|} \sin k\sigma_1 \sin k\sigma_2 \quad (15)$$

If we take a Fourier transform with respect to t , the k -th term in this expression will give $G_k \sim \frac{1}{\omega^2 + k^2}$. The variable t is conjugate to the dilatation operator. It is easy to see that in the case of arbitrary mass, the answer will be

$$G(\omega, \sigma_1, \sigma_2) = \sum_{k=0}^\infty \frac{\Phi_k(\sigma_1)\Phi_k(\sigma_2)}{\omega^2 + (k + \Delta)^2} \quad (16)$$

Here $\Delta = \mu + \frac{1}{2}$ is the lowest dimension which is increased by the raising operators ; the eigenfunctions are expressed through the Legendre functions and form the space of the lowest weight unitary representation of the symmetry group.

Our next task is to perform the Minkowskian continuation. It can be accomplished by taking $t \Rightarrow it$ in the metric (10). It is also possible to take one of the x -s to be imaginary in Poincare's metric. The fact that in the latter case the resulting coordinates don't cover the whole space is of little consequence, since the propagators depend on the invariant distances only. There are, nevertheless, a number of subtleties which we have to discuss. The variable z is given by the formula (12) in which we have to replace $\cosh(t_1 - t_2) \Rightarrow \cos(t_1 - t_2)$. As a result, now $-\infty \leq z \leq \infty$. Hence, we have to deal with antipodal singularity at $z = -1$ in (13) and (15) . We might consider for a moment to replace the Q - function by the P function which doesn't have this singularity. However this choice is a disaster since the P - function blows up at infinity.

Another puzzle is related to the fact that any function of $n \cdot n'$ is periodic in t variable. That means that they are defined on a space with the closed

time-like geodesics and not on the universal covering space which we are primarily interested in. In other words these propagators have singularities not only at $t = t_1 - t_2 = 0$ (if $\sigma_1 = \sigma_2$) but also at $t_1 - t_2 = 2\pi m$.

The resolution of these puzzles follow from the composition principle. It dictates that while changing $\omega \Rightarrow i\omega$ in order to go the Minkowskian AdS (MAdS), we must take the following propagator

$$G(\omega, \sigma_1, \sigma_2) = \sum_{k=0}^{\infty} \frac{\Phi_k(\sigma_1)\Phi_k(\sigma_2)}{\omega^2 - (k + \Delta)^2 + i0} \quad (17)$$

This formula gives the Feynman propagator for the universal covering space, the \widehat{MAdS} . If we are interested in the original space on which t is a periodic variable, we must restrict ω to the integer values. If we attempt to change the $i0$ prescription, say by taking a superposition of $+i0$ and $-i0$ terms for a propagator, we will destroy the composition, since the convolution on the right hand side is divergent (the ω contour is pinched from the opposite sides).

In the coordinate space the function G is not periodic in time, while still having the form (13) away from the singularities. This happens because the $i0$ prescriptions are different at $t = 0$ and $t = 2\pi m$. This is clear already from the expression (15) since the factor $e^{ik|\tau|}$ is not periodic in t . In the first case we have a standard flat space singularity

$$G \sim \log[(t^2 - (\sigma_1 - \sigma_2)^2 - i0)] \quad (18)$$

which, under the action of the Laplace operator, generates a delta function at the coinciding points. At the same time near $t = 2\pi m$ we have

$$G \sim \log[(t - 2\pi m + i0)^2 - (\sigma_1 - \sigma_2)^2] \quad (19)$$

and there is no delta function at these points.

The singularities at $z = -1$ are physically necessary. They occur, according to (12), when $\cos(t_1 - t_2) = \cos(\sigma_1 + \sigma_2)$. All space-time is represented by a strip $0 \leq \sigma \leq \pi$. The above relation means that the two points are connected by a light geodesic which is reflected from ∞ odd number of times. Let us stress again that our prescription requires to sum over all paths lying *inside the strip*. We do not allow them to go to outside. The boundaries of the strip act as perfect mirrors. If the path is a null geodesics, we expect and get singularities whenever $L(P_{xx'}) = 0$.

Let us turn now to the case of the de Sitter space. All we have to do is to interchange space and time in the AdS propagator and take $\mu \Rightarrow i\mu$ (which inverts the sign of M^2). It is also important to remember that in this case the space is periodic since we are dealing with the one-sheeted hyperboloid. As a result, we get the following propagator

$$G(n, n') = Q_{-\frac{1}{2}+i\mu}(n \cdot n' + i0) \quad (20)$$

which satisfies the composition principle. At the same time the standard propagator, proportional to the P function blows up and the composition integral diverges. It is instructive to analyze these propagators in the Poincare coordinates, although they are geodesically incomplete. Their advantage is simplicity. We have the following wave equation

$$(\partial_\tau^2 + p^2 + \frac{M^2}{\tau^2})g_p(\tau, \tau') = \delta(\tau - \tau') \quad (21)$$

Here we made a Fourier transform with respect to space variable σ and introduced the conjugate momentum p . The resulting Green function is given by $G_p(\tau, \tau') = (\tau\tau')^{\frac{1}{2}}g_p(\tau, \tau')$. The solution corresponding to the Bunch-Davies vacuum is given by

$$g_p(\tau, \tau') \sim H_{i\mu}^{(1)}(p\tau_{<})H_{i\mu}^{(2)}(p\tau_{>}) \quad (22)$$

which, in the case of massless particles is simply

$$G_p(\tau, \tau') \sim p^{-1}e^{-ip|\tau-\tau'|} \quad (23)$$

The prefactor p^{-1} would be p^{-d} in d space dimension and is responsible for the scale invariant spectrum of quantum fluctuations. It is obviously the propagator obtained by the sum over paths lying in the *whole plane*. At the same time, the Poincare coordinates are defined on the upper half-plane. Therefore, the paths in the above propagator are allowed to go to the wrong side of infinity. The propagator (20) has a different behavior. It includes paths which are reflected from infinity ($\tau = 0$), but never go beyond it. This results in the Dirichlet propagator in the massless case

$$G_p(\tau, \tau') \sim \frac{1}{p}[e^{-ip|\tau-\tau'|} - e^{-ip(\tau+\tau')}] \quad (24)$$

while in the general case

$$g_p(\tau, \tau') \sim J_{i\mu}(p\tau_{<})H_{i\mu}(p\tau_{>}) \quad (25)$$

The relation of this propagator to the global one (20) follows from the formula

$$(\tau_1 \tau_2)^{\frac{1}{2}} \int dp J_{i\mu}(p\tau_{<}) H_{i\mu}(p\tau_{>}) e^{ipx} = Q_{-\frac{1}{2}+i\mu}(z) \quad (26)$$

where z is given by (12) with $y \Rightarrow i\tau$.

To understand it better, let us consider the spectral decomposition in the global coordinates. In the de Sitter space these coordinates are obtained from the usual polar coordinates, $ds^2 = d\vartheta^2 + \sin^2 \vartheta d\Omega^2$ on a sphere by the analytic continuation $\vartheta \Rightarrow \frac{\pi}{2} + it$ (where $0 \leq \vartheta \leq \pi$ is a polar angle, while the global time $-\infty \leq t \leq \infty$). On a sphere the eigenmodes are given by $\phi_m \sim P_l^m(\cos \vartheta)$ (where l is the angular momentum and m is a magnetic quantum number). As we make the above substitution, we obtain the Bunch-Davies modes, $\phi_m \sim P_l^m(i \sinh t)$, where this time $l = -\frac{1}{2} + i\lambda$. These modes are selected by the condition that they are *non-singular* at $t = -\frac{i\pi}{2}$ (the south pole of the Euclidean sphere). The reason for this requirement is based on the Hartle-Hawking geometry in which the euclidean half-sphere (presumably describing the tunneling creation of the universe) is joined with the Lorentzian half-hyperboloid. It is far from obvious that this centaur is capable of living. We will see in the next section that it is unstable in the sense discussed in the introduction..

Let us discuss other vacua, corresponding to the full hyperboloid. Generally, the eigenmodes satisfy the Klein - Gordon equation

$$\frac{\partial^2}{\partial t^2} \phi + (\lambda^2 + \frac{(m - \frac{d-1}{2})^2 - \frac{1}{4}}{\cosh^2 t}) \phi = 0 \quad (27)$$

(where we returned to $d+1$ dimensional case for the future references). From this equation it is clear that the spectral decomposition of the Green function must contain both discrete and continuous spectra. We can choose the "in" state by $\phi \sim e^{-i\lambda t}$ as $t \rightarrow -\infty$ and, correspondingly the "out" state with the same asymptotic but at $t \rightarrow \infty$. These sets of modes (we also denote them by (\pm)) describe the vacua without particles at the past or future infinities. They give rise to the " Dirichlet" propagator described above.

The "in" or "-" modes (here $d = 1$ again, to simplify notations) are given by $\phi \sim Q_l^m(i \sinh t); l = -\frac{1}{2} + i\lambda$ (these are the standard Jost functions for this problem, which contain only one exponential at $t \rightarrow -\infty$). This function must be understood as an analytic continuation from $t > 0$. The standard definition of the Q function contains a cut from $-\infty$ to 1. As t

becomes negative, we must go to the second sheet through the cut. As a result we have $\phi \sim Q_l^m(i \sinh t) + i\pi P_l^m(i \sinh t)$ for $t < 0$, where the second term compensates the jump across the cut, making ϕ an analytic function. These modes must be complemented with the discrete states, which are given by the same formula but with $0 \leq l \leq m-1$ (and being an integer). The "out" modes are obtained by the *CPT* reflection. The presence of the discrete states reflects the fact that on a hyperboloid there are closed geodesics (ellipses). The open geodesics (hyperbolae) correspond to the continuous spectrum. In the case of the Hartle-Hawking geometry the regularity on the south pole forces us to drop the discrete states, since the Q function, unlike P is singular there. To sum up, we have three types of modes ("vacua"), "in", "out" and E. The "E/out" propagator is given by the Q function and satisfies the composition principle, the E/E propagator is given by the P function and does not; all other propagators are combinations of P and Q . We will also need the expression for the Q - propagator in dS_{d+1} . It is given by

$$G(z) = \text{const}(1 - z^2)^{\frac{d-1}{4}} Q_{-\frac{1}{2}+i\mu}^{\frac{d-1}{2}}(z) \quad (28)$$

where this time $M^2 = \frac{d^2}{4} + \mu^2$ and as before $z = (n \cdot n')$. It is interesting to notice that for even d this expression reduces to the elementary functions. For example, for $d = 2$ the answer is

$$G = \frac{1}{4\pi i} \frac{e^{-i\mu l}}{\sinh l} \quad (29)$$

where $l = \log(z + \sqrt{z^2 - 1})$ is the geodesic distance between the two points. Our convention is that it is real for $z \geq 1$ (time-like separations), imaginary for $-1 \leq z \leq 1$ (moderate space-like separations). For $z < -1$ the geodesic distance becomes complex. This is a peculiar property of the dS space - such points can't be connected by a real geodesics.

5 The composition principle vs. Hartle and Hawking

In a certain sense our proposal for the propagators on eternal manifolds is different from the Hartle-Hawking proposal for the wave functional. Indeed, we could calculate the correlation function for the two points lying on the

equator of the hyperboloid in the following way. The d -dimensional geometry of the equator is given (according to Hartle and Hawking) by the sum over Euclidean geometries bounded by the equator. In the semi-classical approximation this manifold is simply a hemisphere. So, for a free scalar field we expect the wave functional of the form

$$\Psi[\phi(\sigma)] \sim \exp \int d\sigma d\sigma' D(\sigma, \sigma') \phi(\sigma) \phi(\sigma') \quad (30)$$

where the Poisson kernel $D(\sigma, \sigma') = \partial_\perp \partial_\perp G(\sigma, \sigma')$ is expressed through the normal (with respect to the equator) derivatives of the Green function. In the case of a sphere we can always decompose the functional integral in a following way

$$\int_S D\varphi = \int_E D\phi \left(\int_{S_-} D\varphi \right) \left(\int_{S_+} D\varphi \right) \quad (31)$$

where S_\pm are the hemispheres, E is an equator, and the integrals in the brackets are taken with the boundary condition that the bulk fields φ approach a given value ϕ at the equator. We see that the correlation function for the two points on the equator, calculated with Hartle -Hawking wave functional is the same on a sphere and in the de Sitter space. At the same time, the composition principle dictates that this correlation is given by the Q -function in the de Sitter case and by the P -function on a sphere. The difference is that in one case the amplitudes are given by the sum over paths lying in the full lorentzian manifold (a hyperboloid in our case), while in the other the one half of the hyperboloid is replaced by an euclidean sphere.

The natural question to ask in this situation (which was already touched in the introduction) is what is wrong with the standard approach in which we postulate the Bunch- Davies (or the Hartle- Hawking) vacuum from the start and to discard the composition principle.

To answer this question, let us assume for a moment that the Bunch - Davies vacuum is the correct and stable one. That means that the *in* and *out* states coincide. Therefore, if we introduce some interaction, say $\lambda\varphi^4$, we can use the standard Feynman rules. This leads to the incurable infrared divergence even for the massive particles. For example, the first correction to the Green function has the form

$$G^{(1)}(n, n') \sim \lambda \int dn_1 G(n, n_1) G(n_1, n_1) G(n_1, n') \quad (32)$$

where we integrate over the hyperboloid. If we use the P propagator, the integrand contains the interference terms behaving as $z^\nu z^{-\nu-d} \sim z^{-d}$ (where d is the number of space dimensions and $z = (n \cdot n_1)$). Since the measure $dn_1 \sim z^{d-1} dz$, we see that we have a logarithmic divergence $\log z \sim L$ at large separations (L being a geodesic distance) even for the massive particles. Clearly, in the higher orders we will be getting stronger divergences.

To understand the meaning of this phenomenon, let us digress and consider as an analogy the case of a free scalar field in the flat space-time but in a state with arbitrary occupation numbers, $f(p)$, where p is momentum. The Green function in this state is given by

$$G(p, t_1, t_2) = \frac{1}{2\omega(p)} [(1+f)e^{-i\omega|t_1-t_2|} + fe^{i\omega|t_1-t_2|}] \quad (33)$$

where $\omega = \sqrt{p^2 + m^2}$. If we assume for a moment that this state is stable in the above sense (equality of *in* and *out*), the Feynman perturbation theory leads us to a disaster, since in any loop diagram the interference terms proportional to $f(1+f)$ will be *linearly* infrared divergent due to the cancellation of the exponents. This divergence has a simple meaning: the interaction is changing the occupation numbers and creates a non-zero time derivative of $f(p)$. In the case of weak coupling this time derivative satisfies the Boltzmann equation (written here for the φ^3 case to simplify the notations)

$$\frac{\partial f}{\partial t} \sim \int [(1+f(p))f(p_1)f(p_2) - f(p)(1+f(p_1)(1+f(p_2))] \delta(\omega - \omega_1 - \omega_2) \delta(p - p_1 - p_2) dp_1 dp_2 \quad (34)$$

The linear time divergence is absent only if the collision integral in (34) is zero. This is the steady state condition. To recapitulate, without interaction we can have a stable state with arbitrary occupation numbers $f(p)$. This stability is destroyed by any, however weak, interaction, unless the occupation numbers are given by Bose/Fermi distributions.

Analogously, in our problem the linear divergence is a signal that stability and the composition principle are violated. Indeed, if we use the doubled space-time we can easily derive the formula

$$\frac{\partial}{\partial m} G_{++}(n, n') \sim \int dn_1 [G_{++}(n, n_1)G_{++}(n_1, n') - G_{+-}(n, n_1)G_{-+}(n_1, n')] \quad (35)$$

In this integral the linear divergence cancels. The non-zero contribution of the second term (violating the composition) means that the vacuum we are

working with is unstable. The Bunch- Davies (E) vacuum *fails the eternity test* (we define this test as the vanishing of the second term in (35)).

Let us investigate this condition in more details. The presence of the spider diagrams imply that instead of the vacuum state we must look for an excited states with the occupation numbers $\{f(p)\}$ over the Bunch - Davies vacuum. In the small coupling limit these functions satisfy a Boltzmann equation. The structure of this equation in this case is unusual, since it includes particles creation from the vacuum. Leaving the detailed analyzes for another work, we will only present these non-standard terms. They have the form

$$\frac{\partial f}{\partial t} = \int dp_1 dp_2 \delta(p + p_1 + p_2) |A|^2 [(1 + f(p))(1 + f(p_1))(1 + f(p_2))] + O(f)$$

where

$$A \sim \int_0^\infty \frac{d\tau}{\tau^{d+1}} \tau^{3d/2} H_{i\mu}^{(1)}(p_1\tau) H_{i\mu}^{(1)}(p_2\tau) H_{i\mu}^{(1)}(p_3\tau)$$

This term describes the vacuum decay since it is non-zero even when $f = 0$. The terms not explicitly displayed are the standard terms describing the evolution of the one particle states. It seems plausible that the steady state can be reached only when *the gravity from the created excitation screens the "anti-gravity" of the cosmological constant* leaving us with the Friedman-like universe. I hope to discuss these fascinating questions elsewhere.

Let us explore the non-interacting theory in more details. We expand the partial wave of the field

$$\varphi = a\phi + a^+\phi^* \quad (36)$$

where ϕ - s are some solutions of (27) and a, a^+ are the creation and annihilation operators. The vacuum is defined by the $a|vac\rangle = 0$. Different choices of the solutions correspond to the different vacua. The Green functions, $\langle vac|...|vac\rangle$ are given by

$$G_{++}(t_1, t_2) = \phi(t_>)\phi^*(t_<); G_{+-}(t_1, t_2) = \phi(t_2)\phi^*(t_1), etc \quad (37)$$

Substituting these expressions (36) we find the eternity condition

$$\int dt \phi^2(t) = 0 \quad (38)$$

As we mentioned above, the modes corresponding to the E- vacuum are those which remain finite at the south pole and have the form $\phi_E \sim P_\nu^m(\cos \vartheta)$, $\vartheta = \frac{\pi}{2} + it$. Let us notice also that in terms of the conformal time τ ($\tanh t = \cos \tau$) the euclidean south pole corresponds to $\tau \rightarrow i\infty$. That explains why in the Poincare coordinates the E-vacuum requires the Hankel function (22). The integral (38) can be written in the invariant form as $\int_C d\vartheta \sin \vartheta (P_\nu^m(\cos \vartheta))^2$ (the contour C goes from zero to $\frac{\pi}{2}$ and then to $i\infty$). These integrals in can be expressed in terms of the scattering data. Namely, let

$$\phi(t) \rightarrow \alpha(\mu)e^{-i\mu t} + \beta(\mu)e^{i\mu t}, t \rightarrow \infty \quad (39)$$

Using the standard Wronskian identities, we find

$$\int^T dt \phi^2(t) = i\mu \left(\alpha \frac{\partial \beta}{\partial \mu} - \beta \frac{\partial \alpha}{\partial \mu} \right) + 2\alpha\beta T \quad (40)$$

We conclude that for a manifold to be eternal, we must have zero reflection amplitude. Eternity is a rare thing.

We can also explore vacuum stability without doubling of the manifold. For this purpose we introduce the *in/out* and *in/E* Green functions

$$G(n, n') = \langle out | T\varphi(n)\varphi(n') | in \rangle \langle out | in \rangle^{-1} \quad (41)$$

$$G(n, n') = \langle out | T\varphi(n)\varphi(n') | E \rangle \langle out | E \rangle^{-1} \quad (42)$$

where the time ordering is taken with the respect of the time component of n . In the first case we look at the development of the entire de Sitter space - the full hyperboloid. In the second case we are dealing with the centaur - Euclidean hemisphere joined with the Lorentzian hyperboloid along the equator. As we noticed before, they are given by the Q propagator, the only one satisfying the composition principle. This is all we need, we don't have to know what are the *in* and *out* states. Using the formula for the effective action, $e^{iW} = \langle out | in \rangle$ we find the standard formula

$$\frac{\partial W}{\partial \mu} \sim \int dn G(n, n) \quad (43)$$

Of course the Green function at the coinciding points is infinite, but its imaginary part is finite and determines the imaginary part of the effective action (which will have also a trivial infinity - the total volume of the dS space, since the integrand is independent of n). The result is

$$\frac{\partial}{\partial \mu} \text{Im } W \sim (Vol) \text{Im } Q_\nu(1) \quad (44)$$

(where (Vol) is the infinite volume of the whole space). Using the formula

$$Q_{-\frac{1}{2}+i\mu}(z+i0) - Q_{-\frac{1}{2}-i\mu}(z-i0) = i\pi(\tanh \pi\mu - 1)P_{-\frac{1}{2}+i\mu}(z) \quad (45)$$

we find

$$\text{Im } W \sim (Vol) \log(1 + e^{-2\pi\mu}) \quad (46)$$

This method of finding the vacuum decay mimics the derivation of Schwinger of pair production in the strong electric fields. It is interesting that the effective action can also be expressed in terms of the scattering data. Let us consider the effective action W_m for the given angular momentum m . The partial *in/out* Green function is given by

$$G_m(t, t') = \frac{1}{2\mu\alpha_m(\mu)} \phi_m(t_{>}) \chi_m^*(t_{<}) \quad (47)$$

where φ and χ are the Jost functions for the eq. (27) (having the asymptotics $e^{-i\mu t}$ at $t \rightarrow \pm\infty$ respectively , while $\frac{1}{\alpha}$ is the transmission coefficient). The partial effective action satisfies the identity

$$i \frac{\partial W_m}{\partial \mu^2} = \frac{1}{2} \int dt G_m(t, t) \sim \frac{1}{\alpha_m} \int dt \phi_m(t) \chi_m^*(t) \sim \frac{\partial \log \alpha_m(\mu)}{\partial \mu} \quad (48)$$

The last integral was again evaluated using the Wronskians. Collecting all factors , we find a curious formula

$$e^{iW_m} = \frac{1}{\sqrt{\alpha_m}} = \sqrt{T_m} \quad (49)$$

where T_m is the transmission amplitude. We see once again that the non-vanishing imaginary part of the effective action is related to the presence of reflection. The reflection amplitude for the equation (27) can be extracted from the reader's favorite text book on quantum mechanics (the Landau/Lifshits in my case). But one must be careful in comparing the partial wave actions with the complete one (44), since the summation on m diverges and can easily give a wrong result (instead of the infinite invariant volume)

We come to the conclusion that the de Sitter space of even dimensionality is *intrinsically unstable*. Whatever we do, we can't build the eternal space. In the case of the odd dimensions we don't get the imaginary part of the action. The reason for this can be traced to the equation (27). If d is odd , the potential in this Shrodinger equation can be written as $\frac{n(n+1)}{\cosh^2 t}$ with integer n .

This is a soliton of the KdV equation which gives *reflectionless potential* (the vanishing of the reflection coefficient was noticed by the explicit calculations in [9]).

As a side remark let us notice that in the case of general FRW spaces with the scale factor $a(t)$ one may try to look at the more general reflectionless potentials, corresponding to the separated solitons, in an attempt to find eternal backgrounds. This relation between eternal manifolds and completely integrable systems seems quite intriguing.

Returning back to the odd dimensional spaces we come to the following conclusion. If we consider a full de Sitter space, with the past and future infinities it seems that at least in one loop approximation we have a stable manifold. However, if we consider the Hartle - Hawking geometry, the amplitude $\beta \neq 0$ and the stability is lost.

As another side remark, let us notice that in the case of AdS spaces the above analyzes indicates that the simply connected global AdS is stable, but the one with the closed time-like geodesics is not (this is easily seen from the propagators (17) in which ω becomes discrete in the multiply connected case). Such geodesics seem to destabilize a manifold in general by giving an imaginary part to the *in/out* propagators in the formula (1).

6 The infrared effects in the de Sitter space

In this section we will discuss infrared interactions. We already saw how significant they are in a number of examples. It is desirable to have a general estimate of their strength. In the flat space such estimates are well known and very important - they led to the notion of relevant and irrelevant operators. In [1] I tried to evaluate the infrared corrections to the cosmological constant in the Euclidean signature, while in the later work [5] and [7] a number of results were obtained in the in-in formalism.

Let us remember the infrared situation in the flat space, by considering the amplitude for two interacting paths (with φ^4 interaction). The relevance of this interaction is determined by the probability for these two Brownian paths to intersect. The standard estimate is to compare the propagation without interaction which contributes $G^2(R)$ (where $G \sim \frac{1}{R^{D-2}}$ is the Green function and R is a typical distance) with the interaction term which gives $G^4(R)R^D$ (where the second factor is the volume in which the interaction takes place). From here one finds the critical dimension $D_{cr} = 4$.

Let us analyze the de Sitter case in a similar manner. The Bunch - Davies propagator has the following asymptotic behavior

$$G(z - i0) \rightarrow z^{-\frac{d}{2}}(A(\mu)z^\mu + A(-\mu)z^{-\mu}) \quad (50)$$

where $z = (n_1 n_2)$ and $\mu = \sqrt{\frac{d^2}{4} - M^2}$. We have to distinguish the cases of the *light particles* with $M^2 \leq \frac{d^2}{4}$ and the heavy ones with the opposite inequality. In the previous sections we dealt mostly with the heavy particles for which we replaced $\mu \Rightarrow i\mu$. Here we will be interested in both cases. Let us consider the φ^N interaction and begin with the Feynman perturbation theory. Its significance is measured by the amplitude

$$F(n_1 \dots n_N) \sim \int (dn) G(nn_1) \dots G(nn_N) \quad (51)$$

To evaluate convergence of this integral we use the first term in (50) and the fact that at large z (the infrared limit) the measure $(dn) \sim z^{d-1} dz$.

We see that for the sufficiently light particles the integral has a power like divergence. The condition for it is given by

$$N\mu \geq \frac{N-2}{2}d \quad (52)$$

For the φ^4 interaction this gives the condition $M^2 \leq \frac{3d^2}{16}$. The massless case, $\mu = \frac{d}{2}$, is always IR divergent. Because of the $-i0$ prescription, in general these divergent terms have imaginary parts (which may cancel in some special cases).

If the mass is large enough for the above integral to converge, it is easy to see that it is dominated by the region $(n_i n_j) \sim (n_k n)^2 \gg 1$. In this case an easy estimate shows that the interaction is *always marginal*. This can be seen in the Poincare coordinates in which the integral (51) takes the form

$$F \sim (\tau_1 \dots \tau_N)^{\frac{d}{2}-\mu} \int \frac{d^d x d\tau}{\tau^{d+1}} \frac{\tau^{N(\frac{d}{2}-\mu)}}{\prod ((x - x_k)^2 - (\tau - \tau_k)^2)^{\frac{d}{2}-\mu}} \quad (53)$$

In this form (52) is just the divergence (convergence) condition at $\tau \rightarrow 0$. As we compare the value of F at small τ_k with the non-interacting N - point function, we obtain the above result for the convergent case.

In order to study back reaction we need to know various polarization operators. The simplest one appears if we add to the Lagrangian the a term $\int (dn) A(n) J(n)$ where $J(n) = \varphi^2(n)$. As usual we define the back reaction (for the massless case) from the relation

$$\delta J(n) = \int (dn') \Pi(n, n') A(n') = \int (dn') G^2(nn') A(n') \sim \int (dn') \log^2(nn' - i0) A(n') \quad (54)$$

In the case of *in/in* formalism the structure of the infrared divergences is slightly different because of the partial cancellations between the (\pm) Green functions (see also [7]). To evaluate them we need the relations

$$G_{++}(n, n') = g(nn' - i0); G_{+-}(n, n') = g(nn' + i\epsilon \operatorname{sgn}(n_0 - n'_0)) \quad (55)$$

$$G_{--}(n, n') = g(nn' + i0); G_{-+}(n, n') = g(nn' - i\epsilon \operatorname{sgn}(n_0 - n'_0)) \quad (56)$$

The back reaction formula (54) is replaced by

$$\delta J(n) = \int (dn') [G_{++}^2(n, n') - G_{+-}^2(n, n')] A(n') \quad (57)$$

This time the integration goes over the region inside the past light cone (outside the cone $G_{++} = G_{+-}$ and the integrand is zero). Nevertheless we still have the long range effect since $\log^2(z - i0) - \log^2(z + i0) = 2\pi i \log|z|$. Thus

$$\delta J(n) \propto \int (dn') \theta(n_0 - n'_0) \theta(nn' - 1) \log(nn') A(n') \quad (58)$$

The formula (58) must be substituted into the equations of motion for the field A . The resulting effective equations can't be derived from any action principle, since the kernel in (58) is not symmetric with respect to n and n' . The long range correlations should generate the Jeans-like instabilities. However, to make the discussion realistic, we must replace the scalar field with the gravitational fluctuations, fix the gauge and account for the tensor structure. This may change the answer. We leave this for the future work.

The contribution of the heavy particles (for which we have to change $\mu \Rightarrow i\mu$) to the polarization operator is dominated by the interference term. As a result, in the Feynman case, we get the kernel $\sim (nn')^{-d}$. This creates a logarithmic divergence, which we already discussed in connection with the composition principle

7 The gauge/strings duality

Let us discuss now the gauge theory dual of the de Sitter space. It was pointed out in [12] that the natural candidate for it is a gauge theory with the complex coupling constant. We can now present more arguments in favor of this assertion.

Gauge/ string duality is usually understood as a relation between the gauge theory correlators and the string theory correlators in $D+1$ dimension placed at infinity (which is D -dimensional as it should). While it is often convenient, this point of view may cause trouble in the cases when the manifold has a complicated infinity or no infinity at all. Perhaps a more general construction is to consider the isomorphism between the vertex operators of string theory (a set of primary operators with the world sheet dimensions $(1, 1)$) and a set of the field theory operators. In other words we consider a 2d CFT on the world sheet with a critical central charge (the Liouville field is assumed to be included in the menu). We couple it to the world sheet gravity , which (in this case) amounts to selecting the above primaries and integrating them over the world sheet, forming the vertex operators. The resulting objects are identified with the colorless field theory operators and the world sheet OPE are transplanted to the space-time.

The derivation of the gauge/ strings duality from the first principles is a fundamental and unsolved problem. Still, there are a number of indirect qualitative arguments helping to find the duality in each particular case. The simplest one runs as following. Suppose that we have a string theory in the background $ds^2 = d\varphi^2 + a^2(\varphi)dx^2$, where x are the coordinates of the space-time in which a gauge theory is located, while the φ is the Liouville direction. Let us place the Wilson loop at the value $\varphi = \varphi_*$ such that $a(\varphi_*) = \infty$. This Wilson loop defines the open string amplitudes. The slopes of closed and open strings are different. In this case we have $\alpha'_{open} \sim a^{-2}(\varphi_*)\alpha'_{closed} \rightarrow 0$. But in the zero slope limit only the massless states survive, giving the gauge theory. The closed string still has infinite number of states corresponding to the gauge invariant operators, while the open string has finite number of massless states (gluons etc.) The key point of the argument is that the infinite blue shift sends all massive modes of the open string to infinity.

We can now apply this argument to two possible cases. The first case is the centaur geometry (half -sphere, half- hyperboloid). The topology in this case is the same as in the AdS , which has the one component infinity , conformally equivalent to the flat space. Once again, the a factor provides

an infinite blue shift and we can expect that there is a gauge theory in d - dimensions, describing the $d+1$ dimensional strings in the centaur background. The single trace operator of the gauge theory must be equal to the vertex operators, giving the usual relation for the generating functions

$$\log \langle \exp i \int dx \sum_n h_n(x) Tr O_n \rangle = iW[h_n(x, t)] \quad (59)$$

where the $Tr O_n$ are the gauge-invariant operators, while W is the effective action in the h -background for the strings on the centaur. The right hand side is the generating function for the vertex operators. This relation means that we can identify gauge and vertex operators in a following way (in the Poincare coordinates)

$$Tr O_n(x) = \int d^2\xi V_n(x + x(\xi), y(\xi)) \quad (60)$$

where the integration goes along the world sheet and the $V_n(\xi)$ - s are the primary operators of the string sigma model which satisfy the on-shell condition $(L_0 - 1)V_n = 0$. In the low curvature limit this condition reduces to the Klein- Gordon equation. The expectation values $\langle V_{n_1} \dots V_{n_k} \rangle$ are given by the sum of tree diagrams in space-time with the set of massive and massless string states and the legs at infinity. In the AdS case the propagators in this diagrams are given by the $Q_{-\frac{1}{2}+\mu}(z)$ function . To pass to the case of the positive curvature , all we have to do is to change $\mu \Rightarrow i\mu$. As a result we obtain the same tree diagrams but with the propagators $Q_{-\frac{1}{2}+i\mu}$. As we have learned, these propagators correspond to taking matrix elements $\langle out | \dots | E \rangle$ in the centaur geometry. The new piece of information in these formulae is the statement that the analytic continuation from AdS to dS (or , more precisely, to the centaur) leads to the out/E matrix elements, see also [11]).

Let us discuss first the $\mathcal{N}=4$ gauge theory. In the strong coupling limit the change $\mu \Rightarrow i\mu$ corresponds to the change of the gauge coupling $\sqrt{\lambda} = \sqrt{g_{YM}^2 N} \Rightarrow -\sqrt{\lambda}$. This is obviously a non-unitary conformal theory with some of the anomalous dimensions being negative or complex, giving another manifestation of the intrinsic instability of the de Sitter space. Strictly speaking, in this example we have not only the de Sitter background but also the RR fields which become imaginary under the analytic continuation. Gauge theories with the complex coupling are expected to be unstable and so are the corresponding de Sitter spaces. Apart from other things, the change

of the sign of $\sqrt{\lambda}$ at large λ changes the sign of the Coulomb interaction of two charges. In this case one expects Dyson's instability, which potentially can terminate de Sitter's inflation.

So far we discussed the continuation of the maximally supersymmetric case, which is the easiest but not terribly realistic. In the next section we look at the more general situation.

8 De Sitter sigma model

To discuss string theory we need a conformal sigma model on the world sheet with the de Sitter target space. Let us begin with a well studied case of a sphere. It will be convenient for our discussion to discretize one of the world-sheet directions, keeping the other continuous. In this case the hamiltonian of the model can be written as

$$H = \alpha_0 \sum_x l_x^2 + \frac{1}{\alpha_0} \sum_x (n_x - n_{x+1})^2 \quad (61)$$

where α_0 is the bare coupling constant, n is a unit vector, and l is the corresponding angular momentum operator. This model is not conformal and develops a mass gap. Let us remember how to see this (without using the available exact solution of this model). First one calculates the one loop beta function and find that this model is asymptotically free. That means that the coupling grows as we go to the infrared and if the higher orders don't stop it, leads to the mass gap $m^2 \sim \exp(-\frac{const}{\alpha_0})$ at small coupling. If, on the other hand there is a zero of the beta function at some finite value of α , the theory will have no mass gap and will be conformal. To choose between the possibilities we look at the strong coupling limit of (61). The second term can be neglected and the model reduces to a collection of uncoupled rotators. The lowest excitation is obtained by taking $l = 1$ at some site and has a mass gap $\sim \alpha_0$ at large α . We conclude that the beta function has no zeroes and the theory is massive for all couplings. Of course it is possible that the beta function would have two zeroes, but this seems unlikely and indeed, according to the exact solution, doesn't happen.

We see that the outcome depends on two factors - first, the sign of the one-loop beta function which is determined by the sign of the curvature of the target space and the second - compactness/ non-compactness of this space.

The compactness in the above example was responsible for the discreteness of the spectrum and the possibility to drop the last term in (61).

Let us now consider generalizations of this method for dS and AdS spaces and their cosets. Perhaps the simplest case is the one with constant negative curvature but compact target space. This can be realized in two ways. Either one considers the so called $O(n)$ models with $n < 2$ (the curvature of an n -sphere $\sim n - 2$), or one factors AdS space by a discrete subgroup with the compact fundamental domain. In both cases we have positive beta function at the weak coupling and the discrete spectrum in the strong coupling limit. Therefore we expect the beta function *to have a zero at some intermediate coupling*. In the case of the $O(n)$ models these conformal fixed points are well known - they are simply and explicitly described by the minimal models. In the case of the compact cosets the conformal theory is still unknown.

As we return to the de Sitter space, we notice first of all that its beta function coincides with that of a sphere in all orders of perturbation theory. This follows from the fact that the transformation $n_0 \Rightarrow in_0$ taking us from the sphere to the dS space doesn't change the any Feynman diagram. Nevertheless these two theories are very different. We have already mentioned that the angular momentum in the dS space (or in centaur geometry takes the values $l = -\frac{1}{2} + i\lambda$ where λ is real. That means that the spectrum of kinetic energy in the strong coupling limit is continuous and we expect no mass gap. This is a conjecture. What we said so far doesn't really prove it because in the non-compact case we can't simply drop the potential energy, since n can become large. Some variational estimates show that by taking the slow varying n we can suppress the potential energy and end up with the gapless spectrum. Intuitively the continuous spectrum appears because the de Sitter space is non-compact, unlike a sphere. It is highly desirable to have a proof of this statement.

If we believe this conjecture, we must conclude that the dS sigma model has an infrared fixed point which resolves the conflict between the negative beta function at the weak coupling and the gapless phase at the strong. Generally speaking the central charge at this fixed point is smaller the the critical one. However, adding real RR fields to the background allows to adjust it to the critical value.

So we expect that the de Sitter space without any supersymmetry can be described by a string sigma model. This model has a fixed point determined by the zero of the beta function. That means that the curvature of the AdS is fixed. The dual gauge theory must also be at the fixed point. At the moment

we do not have explicit description of these fixed point. We will make the following conjecture about its origin.

In '74 t'Hooft conjectured that planar diagrams become dense and can be described by a string's world sheet. In many recent papers it is stated that AdS/ CFT is a realization of this idea. This statement is wrong because in the usual gauge/ string duality Feynman's diagrams don't become dense at all. The origin of strings in this case are electric flux lines , something quite different from the propagators.

The planar diagrams have a finite radius of convergence and at some complex coupling constant really become dense. It is tempting to relate that to the fixed point of the de Sitter sigma model described above. If this is correct, the " t'Hooft string" describes gauge theory at the critical complex coupling and lives in the de Sitter space. However, at present we don't have the necessary tools to check this conjecture.

So far we discussed the Hartle - Hawking geometry. Next, it is natural to ask what is the gauge theory dual in the complete de Sitter space. The main feature in this case is the existence of two infinities, past and future. It is natural to conjecture that in this case we are dealing with two interacting gauge theories living at these far away locations. The nature of this duality and the possible forms of the trace-trace interactions remains to be clarified.

I would like to thank T. Damour for our conversations over many years if not decades about infrared effects in cosmology. I am very grateful to J. Maldacena for sharing his (different) views on dS space. Discussions with I. Klebanov and N. Nekrasov were , as always, very useful.

Finally, my deep gratitude goes to D. Makogonenko for constant encouragement, advice and moral support.

This work was partially supported by the NSF grant 0243680. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

- [1] A. M. Polyakov, 'Phase Transitions And The Universe,' Sov. Phys. Usp. **25** (1982) 187 [Usp. Fiz. Nauk **136** (1982) 538]

- [2] A. Strominger, “The dS/CFT correspondence,” JHEP **0110** (2001) 034 [arXiv:hep-th/0106113].
- [3] J. S. Schwinger, “Brownian motion of a quantum oscillator,” J. Math. Phys. **2**, 407 (1961).
- [4] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)]
K. T. Mahanthappa, “Multiple production of photons in quantum electrodynamics,” Phys. Rev. **126**, 329 (1962)
- [5] N. C. Tsamis and R. P. Woodard, “Quantum Gravity Slows Inflation,” Nucl. Phys. B **474** (1996) 235 [arXiv:hep-ph/9602315].
- [6] R. Jackiw, C. Nunez and S. Y. Pi, “Quantum relaxation of the cosmological constant,” Phys. Lett. A **347** (2005) 47 [arXiv:hep-th/0502215].
- [7] S. Weinberg, “Quantum contributions to cosmological correlations,” Phys. Rev. D **72**, 043514 (2005) [arXiv:hep-th/0506236]. S. Weinberg, “Quantum contributions to cosmological correlations. II: Can these corrections become large?,” Phys. Rev. D **74**, 023508 (2006) [arXiv:hep-th/0605244].
- [8] E. Witten, “Quantum gravity in de Sitter space,” arXiv:hep-th/0106109.
- [9] R. Bousso, A. Maloney and A. Strominger, Phys. Rev. D **65**, 104039 (2002) [arXiv:hep-th/0112218].
- [10] A. A. Starobinsky, “A new type of isotropic cosmological models without singularity,” Phys. Lett. B **91**, 99 (1980).
- [11] J. M. Maldacena, “Non-Gaussian features of primordial fluctuations in single field inflationary models,” JHEP **0305**, 013 (2003) [arXiv:astro-ph/0210603]
- [12] A. M. Polyakov, “Beyond space-time,” Proceedings of the 23 Solvay conference on physics
arXiv:hep-th/0602011.
- [13] ND Birrel, PCW Davies Quantum Fields in Curved Space-Time - 1984 - Cambridge University Press

- [14] I. Antoniadis, P. O. Mazur and E. Mottola, “Cosmological dark energy: Prospects for a dynamical theory,” *New J. Phys.* **9**, 11 (2007)
- [15] Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper and E. Mottola, “Fermion Pair Production In A Strong Electric Field,” *Phys. Rev. D* **45**, 4659 (1992).
- [16] I. Antoniadis and E. Mottola, ‘4-D quantum gravity in the conformal sector,” *Phys. Rev. D* **45**, 2013 (1992).
- [17] N. P. Myhrvold, *Phys. Rev. D* **28**, 2439 (1983).